Chapter 14

Ten Warning Signs of Algebraic Pitfalls

So much algebra is done in the world, so the sheer number of people who use algebra means that a large number of errors are unavoidable. Some errors occur because that error seems to be an easier way to do the problem. Not right, but easier — the path of least resistance. The main errors in algebra occur while performing expanding-type operations: distributing, squaring binomials, breaking up fractions, or raising to powers. The other big problem area is in dealing with negatives. Watch out for those negative vibes.

Including the Middle Term

A squared binomial has three terms in the answer. The term that often gets left out is the middle term: the part you get when multiplying the two outer terms together and the two inner terms together and finding their sum. The error occurs when just the first and last separate terms are squared, and the middle term is just forgotten.

Right	*Wrong*
$(a + b)^2 = a^2 + 2ab + b^2$	$(a + b)^2 \neq a^2 + b^2$

Keeping Distributions Fair

Distributing a number or a negative sign over two or more terms in parentheses can cause problems if you forget to distribute the outside value over every single term in the parentheses. The errors come in when you stop multiplying the terms in the parentheses before you get to the end.

Right | **Wrong**

$$x - 2(y + z - w) = x - 2y - 2z + 2w \qquad x - 2(y + z - w) \neq x - 2y + z - w$$

Creating Two Fractions from One

Splitting a fraction into several smaller pieces is all right as long as each piece has a term from the numerator (top) and the entire denominator (bottom). You can't split up the denominator.

Right

$$\frac{x + y}{a + b} = \frac{x}{a + b} + \frac{y}{a + b}$$

Wrong

$$\frac{x + y}{a + b} \neq \frac{x}{a} + \frac{y}{b}$$

Restructuring Radicals

If the expression under a radical has values multiplied together or divided, then the radical can be split up into radicals that multiply or divide. You can't split up addition or subtraction, however, under a radical.

Right

$$\sqrt{a^2 + b^2} = \sqrt{a^2 + b^2}$$

Wrong

$$\sqrt{a^2 + b^2} \neq \sqrt{a^2} + \sqrt{b^2}$$

Note: The radical expression is unchanged, because the sum has to be performed before applying the radical operation.

Including the Negative (Or Not)

The order of operations instructs you to raise the expression to a power before you add or subtract. A negative in front of a term acts the same as subtracting, so the subtracting has to be done last. If you want the negative raised to the power, too, then include it in parentheses with the rest of the value.

Right

$-3^2 = -9$
$(-3)^2 = 9$

Wrong

$-3^2 \neq 9$

Making Exponents Fractional

A fractional exponent has the power on the top of the fraction and the root on the bottom.

When writing $\sqrt{x}$ as a term with a fractional exponent, $\sqrt{x} = x^{\frac{1}{2}}$. A fractional exponent indicates that there's a radical involved in the expression. The two in the fractional exponent is on the bottom — the root always is the bottom number.

Right

$\sqrt[5]{x^3} = x^{\frac{3}{5}}$

Wrong

$\sqrt[5]{x^3} \neq x^{\frac{5}{3}}$

Keeping Bases the Same

When you're multiplying numbers with exponents, and those numbers have the same base, you add the exponents and leave the base as it is. The bases never get multiplied together.

Right

$2^3 \cdot 2^4 = 2^7$

Wrong

$2^3 \cdot 2^4 \neq 4^7$

Powering Up a Power

To raise a value that has a power (exponent) to another power, multiply the exponents to raise the whole term to a new power. Don't raise the exponent itself to a power — it's the base that's being raised, not the exponent.

Right

$(x^2)^4 = x^8$

Wrong

$(x^2)^4 \neq x^{16}$

Making Reasonable Reductions

When reducing fractions with a numerator that has more than one term separated by addition or subtraction, then whatever you're reducing the fraction by has to divide every single term evenly in both the numerator and the denominator.

Right

$$\frac{\left(4+6x\right)}{4} = \frac{\left(2+3x\right)}{2}$$

Wrong

$$\frac{\left(4+6x\right)}{4} \neq \frac{\left(2+6x\right)}{2}$$

Catching All the Negative Exponents

When changing fractions to equivalent expressions with negative exponents, give every single factor in the denominator a negative exponent.

Right

$$\frac{1}{2ab^2} = 2^{-1}a^{-1}b^{-2}$$

Wrong

$$\frac{1}{2ab^2} \neq 2a^{-1}b^{-2}$$